5 Linear Inequalities

Linear inequations can be found all around us—we just need to know where to look. Consider the following scenario: On the highway, there are speed limits. When discussing these situations, we frequently use limits, such as "the speed limit is 55 miles per hour." However, we are not required to travel at 55 miles per hour on the highway.

Topic Notes

- *Linear Inequalities and their Algebraic Solutions*

LINEAR INEQUALITIES AND THEIR ALGEBRAIC SOLUTIONS 1

TOPIC 1

INTRODUCTION TO LINEAR INEQUALITY

Two real numbers or two algebraic expression connected by the symbol '<' (less than), '>' (greater than), '≤' (less than or equal) or '≥' (greater than or equal) form an inequation or inequality.

An inequality can be of the following forms:

$a < b$ (read as 'a is less than b')

$a > b$ (read as 'a is greater than b')

$a \leq b$ (read as 'a is less than or equal to b')

$a \geq b$ (read as 'a is greater than or equal to b')

Two real numbers or two algebraic expression related by the symbols '<', '>', '≤' or '≥' form an inequality.

Linear Inequality

An inequality is said to be linear, if each variable occurs in first degree only and there is no term involving the product of the variables.

e.g., $ax + b \leq 0$, $ax + by + c > 0$, $ax \leq 4$.

Solution of an Inequality

Any solution of an inequality is the value(s) of variable(s) which makes it a true statement.

We can find the solutions of an inequality by hit and trial time but it is not very efficient because this method is time consuming and sometimes not feasible. So, we solve inequalities with systematic technique.

Solution Set

The set of all solutions of an inequality is called the solution set of the inequality.

Algebraic Solutions of Linear Inequalities in One Variable

Any solution of an linear inequality in one variable is a value of the variable which makes it a true statement.

eg. $x = 1$ is the solution of the linear inequality $4x + 7 > 0$.

Method to Solve a Linear Inequality in One Variable

Step I: Collect all terms involving the variable (x) on one side and constant terms on other side with the help of above rules and then reduce it in the form $ax < b$ or $ax \leq b$ or $ax > b$ or $ax \geq b$.

Step II: Divide this inequality by the coefficient of variable (x). This gives the solution set of given inequality.

Step III: Write the solution set.

Let us consider the inequality $15x < 200$

Obviously, x cannot be a negative integer or a fraction. Left-hand side (L.H.S) of this inequality is $15x$ and right-hand side (RHS) is 200. Therefore, we have

For $x = 0$, L.H.S. = 15 (0) = 0 < 200 (R.H.S), which is true.

For $x = 1$, L.H.S. = 15 (1) = 15 < 200 (R.H.S), which is true.

For $x = 2$, L.H.S. = 15 (2) = 30 < 200, which is true.

For $x = 3$, L.H.S. = 15 (3) = 45 < 200, which is true.

For $x = 4$, L.H.S. = 15 (4) = 60 < 200, which is true.

For $x = 5$, L.H.S. = 15 (5) = 75 < 200, which is true.

For $x = 6$, L.H.S. = 15 (6) = 90 < 200, which is true.

In the above situation, we find that the values of x, which makes the above inequality a true statement, are 0, 1, 2, 3, 4, 5, 6. These values of x, which make above inequality a true statement, are called solutions of inequality and the set {0, 1, 2, 3, 4, 5, 6} is called its solution set.

As a result, any solution of an inequality in one variable is a variable value, making the proposition true.

We discovered the answer to the aforementioned inequality using a time-consuming trial and error process.

Important

- *Equal numbers can be added (or subtracted) to (from) both sides of an inequality.*
- *A non-zero number can be multiplied (or divided) on both sides of an inequality.*
- *Equal numbers can be added to (or withdrawn from) both sides of an inequality without changing its sign.*
- *An inequality can be multiplied (or divided) by the same positive value on both sides. The sign of inequality is reversed when both sides are multiplied or divided by a negative value.*
- *Any solution of an inequality in one variable is a value of the variable which makes it a true statement.*

Example 1.1: Solve $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$. [NCERT]

Ans. We have, $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

$$\Rightarrow \quad \frac{3x-6}{5} \le \frac{10-5x}{3}$$

$$\Rightarrow \quad 9x - 18 \le 50 - 25x$$

Transposing the terms $(-25x)$ to LHS and the term (-18) to RHS,

$$\Rightarrow \quad 9x + 25x \le 50 + 18$$

$$\Rightarrow \quad 34x \le 68$$

$$\Rightarrow \quad x \le \frac{68}{34}$$

$$\Rightarrow \quad x \le 2$$

$\therefore$ Solution set = $(-\infty, 2]$.

Example 1.2: Solve: $\frac{1}{2}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6)$. [NCERT]

Ans. We have,

$$\frac{1}{2}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{3x}{5}+\frac{4}{1}\right) \ge \frac{1}{3}(x-6)$$

Taking LCM in LHS,

$$\frac{1}{2}\left(\frac{3x+20}{5}\right) \ge \frac{1}{3}(x-6)$$

$$\Rightarrow \quad \frac{3x+20}{10} \ge \frac{x-6}{3}$$

$$\Rightarrow \quad 3(3x+20) \ge 10(x-6)$$

$$\Rightarrow \quad 9x + 60 \ge 10x - 60$$

Transposing the term $10x$ to LHS and the term 60 to RHS,

$$9x - 10x \ge -60 - 60$$

$$\Rightarrow \quad -x \ge -120$$

Multiplying both sides by -1, we get

$$x \le 120$$

$\therefore$ Solution set = $(-\infty, 120]$.

Example 1.3: Solve: $2(2x + 3) - 10 < 6(x - 2)$. [NCERT]

Ans. We have,

$$2(2x+3) - 10 < 6(x-2)$$

$$\Rightarrow \quad 4x + 6 - 10 < 6x - 12$$

$$\Rightarrow \quad 4x - 4 < 6x - 12$$

Transposing the term $6x$ to LHS and (-4) to RHS,

$$\Rightarrow \quad 4x - 6x < -12 + 4$$

$$\Rightarrow \quad -2x < -8$$

On dividing both sides by -2,

$$\Rightarrow \quad \frac{-2x}{-2} > \frac{-8}{-2}$$

$$\Rightarrow \quad x > 4$$

$\therefore$ Solution set = $(4, \infty)$.

Concepts of Intervals on a Number Line

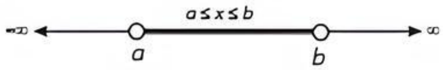
On number line or real line, various types of infinite subsets, known as intervals, are defined below:

Closed Interval

If a and b are real numbers, such that $a < b$, then the set of all real numbers x, such that $a \le x \le b$, is called a closed interval and is denoted by $[a, b]$.

$$\therefore \quad [a, b] = \{x : a \le x \le b, x \in R\}$$

On the number line, $[a, b]$ may be represented as follows:



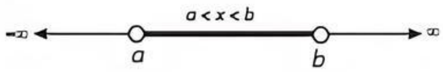
Here, end points of the interval i.e. a and b are included in the interval. So, on number line, draw filled circle (•) at a and b.

Open Interval

If a and b are real numbers, such that $a < b$, then the set of all real numbers x, such that $a < x < b$, is called an open interval and is denoted by (a, b) or $]a, b[$.

$$\therefore \quad (a, b) = \{x : a < x < b, x \in R\}$$

On the number line, (a, b) may be represented as follows:



Here, end points of the interval i.e. a and b are not included in the interval. So, on number line, draw open circle (o) at a and b.

Semi-Open Or Semi-Closed Intervals

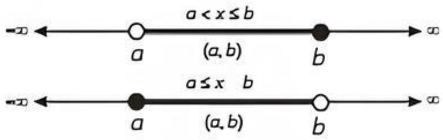
If a and b are real numbers, such that $a < b$.

Then, $(a, b] = \{x : a < x \le b, x \in R\}$

And $[a, b) = \{x : a \le x < b, x \in R\}$

Are known as semi-open or semi-closed intervals.

On the number line, these intervals may be represented as follows:



Representation of Solutions of Linear Inequality in One Variable on Number Line

To represent the solution of a linear inequality in one variable on a number line, use the following rules

(i) To represent $x < a$ (or $x > a$) on a number line, put a circle (o) on the number a and dark the line to the left (or right) of the number a.

(ii) To represent $x \le a$ ($x^2 \ge a$) on a number line, put a dark circle (o) on the number a and dark the line to the left (or right) of the number a.

Example 1.4: Solve $5x - 3 \geq 3x - 5$ and represent the solution on number line. [NCERT]

Ans. We have, $5x - 3 \geq 3x - 5$

Transposing the term $3x$ to LHS and the term (-3) to RHS,

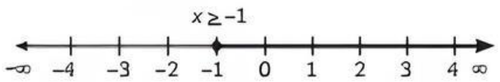
$$5x - 3x \geq -5 + 3$$

$$\Rightarrow \quad 2x \geq -2$$

$$\Rightarrow \quad \frac{2x}{2} \geq -\frac{2}{2}$$

$$\Rightarrow \quad x \geq \frac{-2}{2}$$

$$x \geq -1$$



All the numbers on the right side of -1 will be greater than it.

$\therefore$ Solution set $= [-1, \infty)$.

Example 1.5: Solve $3(1 - x) < 2(x + 4)$ and represent the solution on number line. [NCERT]

Ans. We have, $3(1 - x) < 2(x + 4)$

$$\Rightarrow \quad 3 - 3x < 2x + 8$$

Transposing the term $2x$ to the LHS and the term 3 to RHS,

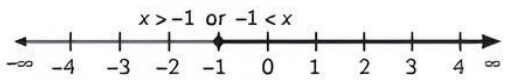
$$-3x - 2x < 8 - 3$$

$$\Rightarrow \quad -5x < 5$$

$$\Rightarrow \quad \frac{-5x}{-5} > \frac{5}{-5}$$

$$\Rightarrow \quad x > \frac{-5}{5}$$

$$\Rightarrow \quad x > -1$$



$\therefore$ Solution set $= (-1, \infty)$.

Example 1.6: A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5cm longer than the second?

[Hint: If x is the length of the shortest board, then x, $(x + 3)$ and $2x$ are the lengths of the second and third piece, respectively. Thus, $x + (x + 3) + 2x \geq 91$ and $2x \geq (x + 3) + 5$]

Ans. Let the length of the shortest board be x cm.

Length of second board 3 cm longer than the shortest side $= x + 3$ cm

and length of the third board = Twice the shortest board $= 2x$ cm

Given, that maximum sum of length of the boards can be 91 cm.

i.e., sum of length of boards ≤ 91

$$x + (x + 3) + 2x \leq 91$$

$$4x + 3 \leq 91$$

$$4x \leq 91 - 3$$

$$4x \leq 88$$

$$x \leq \frac{88}{4}$$

$$x \leq 22$$

Also, the third piece is at least 5 cm longer than the second piece.

$$2x \geq (x + 3) + 5$$

$$2x \geq x + 8$$

$$2x - x \geq 8$$

$$x \geq 8$$

Hence, $x \leq 22$ and $x \geq 8$

i.e., $\quad 8 \leq x \leq 22$

Thus, the possible length of the shortest board is greater than or equal to 8 cm but less than or equal to 22 cm.

TOPIC 2

TYPES OF INEQUALITIES

Strict Inequality

An inequality containing < or > is called strict inequality. A strict inequality can be of the following forms:

$a < b$ or $a > b$

Slack Inequality

An inequality containing ≤ or ≥ is called slack inequality. A slack inequality can be of the following forms:

$a \leq b$ or $a \geq b$

Double Inequality

An inequality containing two simultaneous inequalities is called double inequality. A double inequality can be of the following forms:

$a < b < c$ or $a \leq b < c$ or $a < b \leq c$ or $a \leq b \leq c$

or $a > b > c$ or $a \geq b > c$ or $a > b \geq c$ or $a \geq b \geq c$

Linear Inequality in one Variable

An inequality of the form

$$ax + b < 0 \quad \text{or} \quad ax + b \le 0$$
$$\text{or} \quad ax + b > 0 \quad \text{or} \quad ax + b \ge 0$$

Where $a, b \in R$ $(a \neq 0)$ and x is a real variable, is called linear inequality in one variable x.

Linear Inequality in two Variables

An inequality of the form

$$ax + by + c < 0 \quad \text{or} \quad ax + by + c \le 0$$
$$\text{or} \quad ax + by + c > 0 \quad \text{or} \quad ax + by + c \ge 0$$

Where $a, b, c \in R$ $(a \neq 0, b \neq 0)$ and x, y are real variables, is called linear inequality in two variables x and y.

Quadratic Inequality in one Variable

An inequality of the form

$$ax^2 + bx + c < 0 \quad \text{or} \quad ax^2 + bx + c \le 0$$
$$\text{or} \quad ax^2 + bx + c > 0 \quad \text{or} \quad ax^2 + bx + c \ge 0$$

Where $a, b, c \in R$ $(a \neq 0)$ and x is a real variable, is called quadratic inequality in one variable x.

Important

- ***Solution of an Inequality:*** *The value(s) of the variable(s) that make the inequality a true statement is called the solution of the given inequality.*
- ***Solution set of an Inequality:*** *The set of all possible solutions of an inequality is called the solution set of the given inequality.*

Compound Inequality

An inequality that contains at least two inequalities separated by either 'and' or 'or' is called compound inequalities.

The solution set of a compound inequality with an 'and' represents the intersection of the solution sets of the individual linear inequalities. The compound inequality of the form '$x > a$ and $x < b$' can be expressed as '$x > a$, $x < b$' or '$a < x < b$'.

The solution set of a compound inequality with an 'or' represents the union of the solution sets of the individual linear inequalities. For instance, the compound inequality of the form '$x > 2$ or $x > 3$' can be expressed as '$x > 2$'.

Absolute Value Inequality

Absolute value of a real number: Let x be any real number. Then, the absolute value of x is given by:

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

Clearly, $|x| \ge 0$. Also, $|x|$ represents the distance of x from 0.

For instance, $|-2| = |2| = 2$. Also, both -2 and 2 are two units from zero.

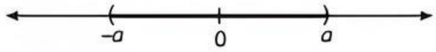
We have the following four types of absolute-value inequalities:

1. $|x| < a$, where a is any positive real number.

   This inequality represents that the distance of x from 0 is less than a.

   Hence, the solution set of given inequality $= \{x \in R: -a < x < a\} = (-a, a)$.

   On the number line $(-a, a)$ may be represented as shown below:

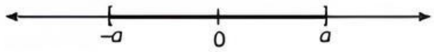


2. $|x| \le a$, where a is any positive real number.

   This inequality represents that the distance of x from 0 is less than or equal to a.

   Hence, the solution set of given inequality $= \{x \in R: -a \le x \le a\} = [-a, a]$.

   On the number line $[-a, a]$ may be represented as shown below:

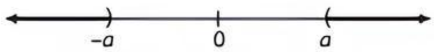


3. $|x| > a$, where a is any positive real number.

   This inequality represents that the distance of x from 0 is greater than a.

   Hence, the solution set of given inequality $= \{x \in R: x < -a \text{ or } x > a\} = (-\infty, -a) \cup (a, \infty)$.

   The representation on the number line of the solution is as shown below:

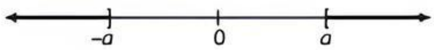


4. $|x| \ge a$, where a is any positive real number.

   This inequality represents that the distance of x from 0 is greater than or equal to a.

   Hence, the solution set of given inequality $= \{x \in R: x \le -a \text{ or } x \ge a\} = (-\infty, -a] \cup [a, \infty)$.

   The representation on the number line of the solution is as shown below:



Example 1.7: **Case Based:**

Reema went to a stationery shop with ₹ 100 to buy notebooks. The price of each notebook is ₹ 25. Let x denotes the number of notebooks.

(A) The inequality which represents the above situation is:

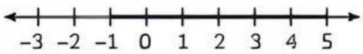
(a) $25x > 100$ (b) $25x \ge 100$
(c) $25x < 100$ (d) $25x \le 100$

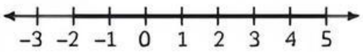
(B) Assertion (A): The maximum number of notebooks that Reema can buy is 4.

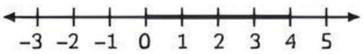
Reason (R): The inequality which represents the situation is $25x > 100$.

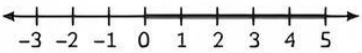
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

(C) The graph representing the above situation is:

(a) 

(b) 

(c) 

(d) 

(D) If Reema gets discount of ₹ 5 on each notebook, then find the maximum number of notebooks that she can buy.

(E) If Reema gets ₹ 25 extra from her mother, then find the inequality representing the given situation.

Ans. (A) *(d) $25x \le 100$*

Explanation: Here, x denotes the number of notebooks.

Cost of one notebook = ₹ 25

Total amount spent by her = ₹ $25x$

As she has the total amount of ₹ 100

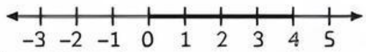
$\therefore$ Required inequality is $25x \le 100$.

(B) *(c) (A) is true but (R) is false.*

Explanation: We have, $25x \le 100$

$\Rightarrow$ $x \le 4$, where x is the number of notebooks

$\therefore$ The maximum number of notebooks that Reema can buy is 4.

(C) *(c)* 

Explanation: As, x represent the number of notebooks

$\therefore$ $x \ge 0$ ...(i)

Also, $x \le 4$ ...(ii)

Combining (i) and (ii), we get

$0 \le x \le 4$

(D) If Reema gets discount of ₹ 5 on each notebook, then the cost of one notebook = ₹ 20

Also, $20x \le 100$

$\Rightarrow$ $x \le 5$

$\therefore$ The maximum number of notebooks that Reema can buy is 5.

(E) If Reema gets ₹ 25 extra from her mother, then she has the total amount of ₹ 125.

Required inequality is $25x \le 125$.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. If $|3 - 6x| \ge 9$, then $x \in$:

(a) $(-\infty, -1) \cup (3, \infty)$ (b) $(-\infty, 1] \cup (2, \infty)$
(c) $(-\infty, -1) \cup (0, \infty)$ (d) $(-\infty, -1) \cup (2, \infty)$

Ans. *(d) $(-\infty, -1) \cup (2, \infty)$*

Explanation: We have, $|3 - 6x| \ge 9$

$\Rightarrow$ $3 - 6x \le -9$ or $3 - 6x \ge 9$

$\Rightarrow$ $-6x \le -12$ or $-6x \ge 6$

$\Rightarrow$ $x \ge 2$ or $x \le -1$

$\Rightarrow$ $(-\infty, -1) \cup (2, \infty)$

2. If x is a real number and $|x| < 3$, then x lies between:

(a) $x \ge 3$ (b) $x \le -3$
(c) $-3 \le x \le 3$ (d) $-3 < x < 3$ [Diksha]

Ans. *(d) $-3 < x < 3$*

Explanation: Given, $|x| < 3$

Here, x is a real number

So,

$x < 3$

$-x < 3$

$\Rightarrow$ $x > -3$

$x \in (-3, 3)$

3. If $2x - 1 < 6 + x$, $4 - 3x \le 1$, then $x \in$:

(a) $[1, 7]$ (b) $[-1, 7]$
(c) $[1, 7)$ (d) $(1, 7)$

Ans. *(c) $[1, 7)$*

Explanation: We have, $2x - 1 < 6 + x$

$2x - x < 6 + 1$

$\Rightarrow$ $x < 7$

Also, we have $4 - 3x \le 1$

$\Rightarrow$ $-3x \le -3$

$\Rightarrow$ $x \ge 1$

Thus, solution of the given system are all real number lying between 1 and 7 including 1, i.e., $1 \le x < 7$.

$\therefore$ $x \in [1, 7)$

4. If $-8 \le 5x - 3 < 7$, then $x \in$:

(a) $(-1, 2)$ (b) $[-1, 2)$
(c) $[-2, \infty)$ (c) $[-2, 0)$

Ans. *(b) [–1, 2)*

Explanation: We have, $-8 \le 5x - 3 < 7$

Adding 3 all sides

$$-8 + 3 \le 5x - 3 + 3 < (7 + 3)$$
$$-5 \le 5x < 10$$

Divide 5 all sides

$$\frac{-5}{5} \le \frac{5x}{5} < \frac{10}{5}$$
$$-1 \le x < 2$$

Thus, x is a real number which is less than 2 and greater than or equal to – 1.

Hence, $x \in [-1, 2]$ is the solution.

5. If $-3 \le \frac{5-3x}{2} \le 4$, then x:

(a) $\left[1, \frac{11}{3}\right]$ (b) [–5, 5]

(c) $\left[-\frac{11}{3}, \infty\right]$ (d) [–∞, ∞]

Ans. (a) $\left[1, \frac{11}{3}\right]$

Explanation: We have,

$$-3 \le \frac{5-3x}{2} \le 4$$
$$\Rightarrow \quad -6 \le 5 - 3x \le 8$$
$$\text{or} \quad -11 \le -3x \le 3$$
$$\Rightarrow \quad \frac{11}{3} \ge x \ge 1, \text{ which can be written as}$$
$$\frac{11}{3} \ge x \ge 1.$$
$$\therefore \quad x \in 1, \frac{11}{3}$$

6. The given inequality for the real x : $4x + 3 < 5x + 7$. The solution set is:

(a) (4, ∞) (b) (2, ∞)

(c) (–4, ∞) (d) (–2, ∞) [Diksha]

Ans. *(c) (–4, ∞)*

Explanation: We have,

$$4x + 3 < 5x + 7$$
$$\Rightarrow \quad 3 - 7 < 5x - 4x$$
$$-4 < x$$
$$\text{or,} \quad x > -4$$
$$\therefore \quad x \in (-4, \infty).$$

7. If $3x - 1 < 5 + x$, $6 - 5x \le 1$, then $x \in$:

(a) [1, 3] (b) [– 1, 3]

(c) [1, 3) (d) (1, 3)

Ans. *(c) [1, 3)*

Explanation: We have,

$$3x - 1 < 5 + x$$
$$\Rightarrow \quad 3x - x < 5 + 1$$
$$2x < 6$$
$$x < 3$$

Also, we have $6 - 5x \le 1$

$$\Rightarrow \quad -5x \le 1 - 6$$
$$\Rightarrow \quad -5x \le -5$$
$$\Rightarrow \quad x \ge 1$$

Thus, solution of the given system are all real number lying between 1 and 3 including 1, i.e.,

$$1 \le x < 3.$$
$$\therefore \quad x \in [1, 3)$$

8. If $\frac{x+3}{x+5} > 3$, then $x \in$:

(a) (–6, 5) (b) (–6, –∞)

(c) (–6, ∞) (d) (6, 12)

Ans. *(b) (–6, –∞)*

Explanation: We have, $x + 3 > 3x + 15$

$$3 - 15 > 3x - x$$
$$\Rightarrow \quad -12 > 2x$$
$$\Rightarrow \quad -6 > x$$
$$\Rightarrow \quad x < -6$$

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

9. **Assertion (A):** If $a < b$, $c < 0$ then $\frac{a}{c} < \frac{b}{c}$.

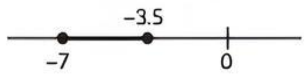
Reason (R): If both sides are divided by the same negative quantity, then the inequality is reversed.

Ans. *(d) (A) is false but (R) is true.*

Explanation: If both sides are divided by the same negative quanaity, then the inequality is reversed. If $a < b$, $c < 0$ then $\frac{a}{c} > \frac{b}{c}$.

10. **Assertion (A):** If $-5 \le 2x + 9 \le 2$, then $x \in [-7, -3.5]$.

Reason (R): The representation on the number line of $-5 \le 2x + 9 \le 2$ is



Ans. (a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: We have $-5 \le 2x + 9 \le 2$

$\Rightarrow \quad -5 - 9 \le 2x \le 2 - 9$

$\Rightarrow \quad -14 \le 2x \le -7$

$\Rightarrow \quad -7 \le x \le \frac{-7}{2}$

$\therefore \quad x \in \left[-7, \frac{-7}{2}\right]$

11. **Assertion (A):** If $11x - 9 \le 68$, then $x \in (-\infty, 7)$.

Reason (R): If an inequality consists of sign $\le$ or $\ge$, then the point on the line are also included in the solution region.

Ans. (d) *(A) is false but (R) is true.*

Explanation: We have,

$11x - 9 \le 68$,

$\Rightarrow \quad 11x \le 77$

$\Rightarrow \quad x \le 7$

$\therefore \quad x \in (-\infty, 7]$

So, assertion is false but the reason is true.

12. **Assertion (A):** $|3x - 5| > 9$

$\Rightarrow \quad x \in \left(-\infty, \frac{-4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$.

Reason (R): The reason containing all the solutions of an inequality is called the solution region.

Ans. (b) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: We have $|3x - 5| > 9$

$\Rightarrow \quad 3x - 5 < -9$ or $3x - 5 > 9$

$\Rightarrow \quad 3x < -4$ or $3x > 14$

$\Rightarrow \quad x < \frac{-4}{3}$ or $x > \frac{14}{3}$

$\therefore \quad x \in \left(-\infty, \frac{-4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

13. *Two real numbers or two algebraic expressions related by the symbols $<, \le, \ge, >$ form an inequation. If the highest power of the variables used in the inequation is 1, then the inequation is called linear inequation.*

(A) If $-3x + 17 < -13$, then:

(a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$
(c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10)$

(B) Given that x, y and b are real numbers and $x < y$, $b < 0$, then:

(a) $\frac{x}{b} < \frac{y}{b}$ (b) $\frac{x}{b} \le \frac{y}{b}$
(c) $\frac{x}{b} > \frac{y}{b}$ (d) $\frac{x}{b} \ge \frac{y}{b}$

(C) If $|x - 1| > 5$, then:

(a) $x \in (-4, 6)$
(b) $x \in [-4, 6]$
(c) $x \in (-\infty, -4) \cup (6, \infty)$
(d) $x \in [-\infty, -4) \cup [6, \infty)$

(D) If $\frac{|x-7|}{(x-7)} \ge 0$, then:

(a) $x \in [7, \infty)$ (b) $x \in (7, \infty)$
(c) $x \in (-\infty, 7)$ (d) $x \in (-\infty, 7]$

(E) If $|x + 3| \ge 10$, then:

(a) $x \in (-13, 7]$
(b) $x \in (13, 7]$
(c) $x \in (-\infty, -13] \cup [7, \infty)$
(d) $x \in [-\infty, -13] \cup [7, \infty)$

Ans. (A) (a) $x \in (10, \infty)$

Explanation: Given,

$-3x + 17 < -13$

Subtracting 17 from both sides,

$-3x + 17 - 17 < -13 - 17$

$\Rightarrow \quad -3x < -30$

$\Rightarrow x > 10$ {since the division by negative number inverts the inequality sign}

$\Rightarrow x \in (10, \infty)$

(B) (a) $\frac{x}{b} < \frac{y}{b}$

Explanation: Given that x, y and b are real numbers and $x < y$, $b < 0$.

Consider, $x < y$

Divide both sides of the inequality by "b"

$\frac{x}{b} < \frac{y}{b}$ {since $b < 0$}

(C) (c) $x \in (-\infty, -4) \cup (6, \infty)$

Explanation: $|x - 1| > 5$

$x - 1 < -5$ and $x - 1 > 5$

$x < -4$ and $x > 6$

Therefore, $x \in (-\infty, -4) \cup (6, \infty)$

(D) *(b)* $x \in (7, \infty)$

Explanation: Given,

$$\frac{|x-7|}{(x-7)} \geq 0$$

This is possible when $x - 7 \geq 0$, and $x - 7 \neq 0$.

Here, $x \geq 7$ but $x \neq 7$

Therefore, $x > 7$, i.e. $x \in (7, \infty)$.

(E) *(d)* $x \in [-\infty, -13] \cup [7, \infty)$

Explanation: Given,

$|x + 3| \geq 10$

$\Rightarrow x + 3 \leq -10$ or $x + 3 \geq 10$

$\Rightarrow \quad x \leq -13$ or $x \geq 7$

$\Rightarrow \quad x \in (-\infty, -13] \cup [7, \infty)$

14. *Amit's mother gave him ₹ 200 to buy some packets of rice and maggie from the market. The cost of one packet of rice is ₹ 30 and that of one packet of maggie is ₹ 20. Let x denotes the number of packet of rice and y denotes the number of packets of maggie.*

(A) Find the inequality that represents the given situation.

(B) If he buys 4 packets of rice and spends entire amount of Rs 200, then find the maximum number of packets of maggie that he can buy.

(C) Solve the following inequality for real x.

$$4x + 3 < 5x + 7$$

Ans. (A) Total amount = ₹ 200

Cost of one packet of rice = ₹ 30

And cost of one packet of maggie = ₹ 20

Here, x and y denote the number of packets of rice and maggie respectively.

Total amount spent by Amit is $30x + 20y$.

$\therefore$ Required inequality is $30x + 20y \leq 200$

(B) If he spends his entire amount, then

We have, $30x + 20y = 200$ ...(i)

Since, number of packet of rice = 4

$\therefore$ At $x = 4$, equation (i) becomes

$30 \times 4 + 20y = 200$

$\Rightarrow \quad 120 + 20y = 200$

$\Rightarrow \quad 20y = 200 - 120$

$\Rightarrow \quad 20y = 80$

$\therefore$ Maximum number of packets of maggie that he can buy is 4.

(C) Given that, $4x + 3 < 5x + 7$

Now by subtracting 7 from both the sides, we get

$4x + 3 - 7 < 5x + 7 - 7$

The above inequality becomes,

$4x - 4 < 5x$

Again, by subtracting $4x$ from both the sides,

$4x - 4 - 4x < 5x - 4x$

$x > -4$

$\therefore$ The solutions of the given inequality are defined by all the real numbers greater than -4.

The required solution set is $(-4, \infty)$.

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

15. Solve the following inequality:

$$10x + 4 < 5x + 9.$$

Ans. We have, $10x + 4 < 5x + 9$

$\Rightarrow \quad 10x + 4 - 5x < 5x + 9 - 5x$

[On subtracting $5x$ from both sides]

$\Rightarrow \quad 5x + 4 < 9$

$\Rightarrow \quad 5x + 4 - 4 < 9 - 4$

[On subtracting 4 from both sides]

$\Rightarrow \quad 5x < 5$

$\Rightarrow \quad \frac{5x}{5} < \frac{5}{5}$

[On dividing both sides by 5]

$\Rightarrow \quad x < 1$

Hence, the solution set of given inequality

$= \{x \in R : x < 1\} = (-\infty, 1)$.

16. Solve the following equation:

$$3x + 17 \leq 2(1 - x).$$

Ans. We have,

$3x + 17 \leq 2(1 - x)$

$\Rightarrow \quad 3x + 17 \leq 2 - 2x$

$\Rightarrow \quad 3x + 2x \leq 2 - 17$

[$\because$ Transposing $-2x$ to LHS and 17 to RHS]

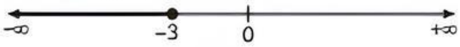
$\Rightarrow \quad 5x \leq -15$

$\Rightarrow \quad \frac{5x}{5} \leq \frac{-15}{5}$

$\Rightarrow \quad x \leq -3$

$\Rightarrow \quad x \in (-\infty, -3]$

Hence, the solution set of the given in equation is $(-\infty, -3]$, which can be represent on real line as shown in figure.



17. **$4x + 3 \geq 2x + 17$, $3x - 5 < -2$.**

[NCERT Exemplar]

Ans. We have, $4x + 3 \geq 2x + 17$

$\Rightarrow \quad 4x - 2x \geq 17 - 3$

$\Rightarrow \quad 2x \geq 14$

$\Rightarrow \quad x \geq \frac{14}{2}$

$\Rightarrow \quad x \geq 7$ ...(i)

Also, we have $3x - 5 < -2$

$\Rightarrow \quad 3x < -2 + 5$

$\Rightarrow \quad 3x < 3$

$\Rightarrow \quad x < 1$ ...(ii)

On combining eq. (i) and (ii) we see that solution is not possible because nothing is common between these two solutions. (i.e., $x < 1$, $x \geq 7$). Thus, it has no solution.

18. **A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$, respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit?**

[NCERT Exemplar]

Ans. We have, Cost function, $C(x) = 26000 + 30x$ and revenue function, $R(x) = 43x$

For profit,

Revenue > Cost

or, $\quad C(x) < R(x)$

$\Rightarrow \quad 26000 + 30x < 43x$

$\Rightarrow \quad 30x - 43x < -26000$

$\Rightarrow \quad -13x < -26000$

$\Rightarrow \quad 13x > 26000$

$\Rightarrow \quad x > \frac{26000}{13}$

$\therefore \quad x > 2000$

Hence, more than 2000 cassettes must be produced to get profit.

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

19. **Solve $|3x - 2| \leq \frac{1}{2}$.**

[Delhi Gov. Term-2 SQP 2022]

Ans. Given, $|3x - 2| \leq \frac{1}{2}$

$\Rightarrow \quad \frac{-1}{2} \leq (3x - 2) \leq \frac{1}{2}$

$[\because |x| \leq a \Rightarrow -a \leq x < a]$

$\Rightarrow \quad \frac{-1}{2} + 2 \leq 3x - 2 + 2 \leq \frac{1}{2} + 2$

[adding 2 on each term]

$\Rightarrow \quad \frac{3}{2} \leq 3x \leq \frac{5}{2}$

$\Rightarrow \quad \frac{3}{2} \times \frac{1}{3} \leq 3x \frac{1}{3} \leq \frac{5}{2} \times \frac{1}{3}$

[dividing each term by 3]

$\Rightarrow \quad \frac{1}{2} \leq x \leq \frac{5}{6}$ *i.e.,* $x \in \left[\frac{1}{2}, \frac{5}{6}\right]$

$|3x - 2| \leq \frac{1}{2}$

$3x - 2 \leq \frac{1}{2}$ or $3x - 2 \geq \frac{-1}{2}$

$x \leq \frac{5}{6}$ or $x \geq \frac{1}{2}$

$\therefore \quad \frac{1}{2} \leq x \leq \frac{5}{6}$

20. **Solve $-15x > 45$, when**

(A) x is a natural number.

(B) x is an integer.

Ans. We have, $-15x > 45$

$\Rightarrow \quad \frac{-15x}{-15} < \frac{45}{-15}$

[On dividing both sides by -15]

$\Rightarrow \quad x < -3$

(A) Given that x is a natural number, i.e., $x \in N$.

Hence, the solution set of given inequality $= \{x \in N : x < -3\}$ = No Solution.

(B) Given that x is an integer, i.e., $x \in Z$.

Hence, the solution set of given inequality $= \{x \in Z : x < -3\} = \{......, -5, -4\}$.

21. **Solve $40x < 100$. when**

(A) x is a natural number.

(B) x is an integer.

Ans. We have, $40x < 100$

$$\Rightarrow \quad \frac{40x}{40} < \frac{100}{40}$$

[On dividing both sides by 40]

$$\Rightarrow \quad x < \frac{5}{2}$$

(A) Given that x is a natural number, i.e., $x \in N$.

Hence, the solution set of given inequality

$$= \left\{x \in N : x < \frac{5}{2}\right\} = \{1, 2\}$$

(B) Given that x is an integer, i.e., $x \in Z$.

Hence, the solution set of given inequality

$$= \left\{x \in Z : x < \frac{5}{2}\right\} = \{......, -2, -1, 0, 1, 2\}.$$

22. **The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, then find the range of pH value for the third reading that will result in the acidity level being normal.**

[NCERT Exemplar, Delhi Gov. QB 2022]

Ans. Given data, first pH value = 8.48 and second pH value = 8.35

Let third pH value be x.

Since, it is given that average pH value lies between 8.2 and 8.5.

$$\therefore \quad 8.2 < \frac{8.48 + 8.35 + x}{3} < 8.5$$

$$\Rightarrow \quad 8.2 < \frac{16.83 + x}{3} < 8.5$$

$$\Rightarrow \quad 3 \times 8.2 < 16.83 + x < 8.5 \times 3$$

$$\Rightarrow \quad 24.6 < 16.83 + x < 25.5$$

$$\Rightarrow \quad 24.6 - 16.83 < x < 25.5 - 16.83$$

$$\Rightarrow \quad 7.77 < x < 8.67$$

Thus, third pH value lies between 7.77 and 8.67.

23. **Find $x^2 + 4ax + 4 > 0$ for all x.** **[Diksha]**

Ans. Given, $\quad x^2 + 4ax + 4 > 0$

$$\Rightarrow \quad x^2 + 4ax + 4a^2 + 4 - 4a^2 > 0$$

$$\Rightarrow \quad (x + 2a)^2 + (4 - 4a^2)^2 > 0$$

$$\Rightarrow \quad 4 - 4a^2 > 0$$

$$\Rightarrow \quad 4 > 4a^2$$

$$\Rightarrow \quad 1 > a^2$$

$$\Rightarrow \quad |a| < 1 - 1 < a < 1$$

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

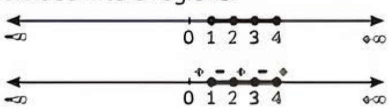
24. **Solve: $\frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0,\ x \in R.$**

[Delhi Gov. QB 2022]

Ans. We have,

$$\frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0$$

Equating $(x - 3)(x - 4)$ and $(x - 1)(x - 2)$ to zero we obtain x = 3, 4, 1, 2 as critical points. Plot these points on the real line as shown below. The real line is divided into 6 regions.



When $x > 4$

$$\therefore \quad \frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0$$

When $3 < x < 4$

$$\therefore \quad \frac{(x-1)(x-2)}{(x-3)(x-4)} \leq 0$$

When $2 \leq x < 3$

$$\therefore \quad \frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0$$

When $1 \leq x \leq 2$

$$\therefore \quad \frac{(x-1)(x-2)}{(x-3)(x-4)} \leq 0$$

When $1 < x < 0$

$$\therefore \quad \frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0$$

When $x < 0$

$$\therefore \quad \frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0$$

Hence solution set,

$$(-\infty, 1] \cup [2, 3]\ (4, \infty)$$

Hence, solution set :

$$\geq 4 \in [4, \infty)$$

25. **Solve: $-5 \leq \frac{2-3x}{4} \leq 9.$** **[NCERT Exemplar]**

Ans. We have, $-5 \leq \frac{2-3x}{4} \leq 9$

Now,

$$\Rightarrow \quad -5 \times 4 \le \frac{2-3x}{4} \le 9 \times 4$$

[Multiplying throughout by 4]

$$\Rightarrow \quad -20 \le 2-3x \le 36$$

$$\Rightarrow \quad -20-2 \le 2-3x-2 \le 36-2$$

[Subtracting 2 throughout]

$$\Rightarrow \quad -22 \le -3x \le 34$$

$$\Rightarrow \quad \frac{-22}{-3} \ge \frac{-3x}{-3} \ge \frac{34}{-3}$$

[Dividing throughout by – 3]

$$\Rightarrow \quad \frac{22}{3} \ge x \ge \frac{-34}{3}$$

$$\Rightarrow \quad \frac{-34}{3} \le x \le \frac{22}{3}$$

$$\Rightarrow \quad x \in \left[\frac{-34}{3}, \frac{22}{3}\right]$$

Hence, the interval $\left[\frac{-34}{3}, \frac{22}{3}\right]$ is the solution set of the given system of in equations.

26. Solve: $-11 \le 4x - 3 \le 13$.

Ans. We have,

$$-11 \le 4x-3 \le 13$$

$$\Rightarrow \quad -11 \le 4x-3 \text{ and}$$

$$4x-3 \le 13$$

Thus, we have two inequalities and we wish to solve them simultaneously. Instead of solving these inequalities by using the method discussed in the first three examples, let us solve them directly in a different way as given below.

We have,

$$-11 \le 4x-3 \le 13$$

$$\Rightarrow \quad -11+3 \le 4x-3+3 \le 13+3$$

[Adding 3 throughout]

$$\Rightarrow \quad -8 \le 4x \le 16$$

$$\Rightarrow \quad \frac{-8}{4} \le x \le \frac{16}{4} \text{ [Dividing by 4 throughout]}$$

$$\Rightarrow \quad -2 \le x \le 4$$

$$\Rightarrow \quad x \in [-2, 4]$$

Hence, the interval [–2, 4] is the solution set of the given system of in equations.

27. Solve the following system of inequalities $\frac{2x+1}{7x-1} > 5, \frac{x+7}{x-8} > 2.$

[NCERT Exemplar, Delhi Gov. QB 2022]

Ans. The given system of inequalities is:

$$\frac{2x+1}{7x-1} > 5 \quad \text{...(i)}$$

and $$\frac{x+7}{x-8} > 2 \quad \text{...(ii)}$$

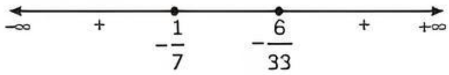
Now, $$\frac{(2x+1)-5(7x-1)}{7x-1} > 0$$

$$\Rightarrow \quad \frac{2x+1-35x+5}{7x-1} > 0$$

$$\Rightarrow \quad \frac{-33x+6}{7x-1} > 0$$

$$\Rightarrow \quad \frac{33x-6}{7x-1} < 0$$

$$\Rightarrow \quad x \in \left(\frac{1}{7}, \frac{6}{33}\right) \quad \text{...(iii)}$$



and $$\frac{x+7}{x-8} > 2$$

$$\Rightarrow \quad \frac{x+7}{x-8} - 2 > 0$$

$$\Rightarrow \quad \frac{x+7-2(x-8)}{x-8} > 0$$

$$\Rightarrow \quad \frac{x+7-2x+16}{x-8} > 0$$

$$\Rightarrow \quad \frac{-x+23}{x-8} > 0$$

$$\Rightarrow \quad \frac{x-23}{x-8} < 0$$

In the figure (–23 should be 23)

$\Rightarrow x \in (8, 23)$ ...(iv)

Since, the intersection of eq. (iii) and (iv) is the null set. Hence, the given system of equation has no solution.

28. Solve the following system of in equations: $|x-1| \le 5, |x| \ge 2$ [NCERT Exemplar]

Ans. The given system of inequalities is

$$|x-1| \le 5 \quad \text{...(i)}$$

$$|x-1| \ge 2 \quad \text{...(ii)}$$

Now, $|x-1| \le 5$ $\quad [\because |x-1| \le r \Leftrightarrow a-r \le a+r]$

Thus, the solution set of (i) is the interval

$$x \in [-4, 6]$$

And, $|x| \ge 2$

$$\Rightarrow \quad x \le -2, \text{ or } x \ge 2$$

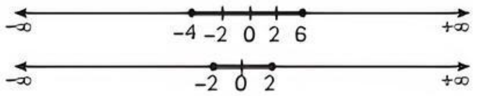
$$[\because |x| \ge a$$

$$\Rightarrow \quad x \le -a \text{ or } x \le a]$$

$$\Leftrightarrow \quad x \in (-\infty, -2] \cup [2, \infty).$$

Thus, the solution set of (ii) is $(-\infty, -2] \cup [2, \infty)$.

The solution sets of inequalities (i) and (ii) are represented graphically in figure (i) and (ii) respectively. The interaction of these two is $[-4, -2] \cup [2, 6]$



Hence, the solution set of the given system of in equations is $[-4, -2] \cup [2, 6]$.

29. Solve: $\frac{2x-3}{4}+9 \geq 3+\frac{4x}{3}$, $x \in R$. **[Delhi Gov. QB 2022]**

Ans. We have,

$$\Rightarrow \quad \frac{2x-3}{4}+9 \geq 3+\frac{4x}{3}$$

$$\Rightarrow \quad \frac{2x-3}{4}-\frac{4x}{3} \geq 3-9$$

$$\Rightarrow \quad \frac{6x-9-16x}{12} \geq -6$$

$$\Rightarrow \quad -10x-9 \geq -72$$

$$\Rightarrow \quad 10x \geq -72+9$$

$$\Rightarrow \quad -10x \geq -63$$

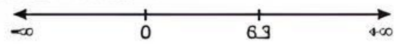
$$\Rightarrow \quad x \leq \frac{63}{10}$$

$$\Rightarrow \quad x \in \left(-\infty, \frac{63}{10}\right)$$

Hence, the solution set of the given inequation is

$x \in \left(-\infty, \frac{63}{10}\right)$.

The representation of solution set on number line is shown below.



LONG ANSWER Type Questions (LA)

[4 & 5 marks]

30. Solve: $\frac{2x-1}{3} \geq \frac{3x-2}{4}-\frac{2-x}{5}$. **[Delhi Gov. QB 2022]**

Ans.

$$\frac{2x-1}{3} \geq \frac{(3x-2)}{4}-\frac{(2-x)}{5}$$

$$\frac{(2x-1)}{3} \geq \frac{5(3x-2)-4(2-x)}{4(5)}$$

$$\frac{(2x-1)}{3} \geq \frac{15x-10-8+4x}{20}$$

$$\frac{(2x-1)}{3} \geq \frac{15x+4x-10-8}{20}$$

$$\frac{(2x-1)}{3} \geq \frac{19x-18}{20}$$

$$20(2x-1) \geq 3(19x-18)$$

$$40x-20 \geq 57x-54$$

$$40x-57x \geq -54+20$$

$$-17x \geq -34$$

$$-x \geq \frac{-34}{17}$$

$$-x \geq -2$$

Since x is negative, we multiply both sides by -1 & change the signs

$$(-1) \times (-x) \leq (-1) \times (-2)$$

$$x \leq 2$$

Hence, x is a real number which is less than or equal to w.

Hence, $x \in (-\infty, 2]$ is the solution.

31. **A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460L of the 9% solution, how many litres of 3% solution will have to be added?**

[NCERT Exemplar, Delhi Gov. Term-2 SQP 2022]

Ans. Let x L of 3% solution be added to 460 L of 9% solution of acid.

Then, the total quantity of mixture = $(460 + x)$ L

Total acid content in the $(460 + x)$ L of mixture

$$= \left(460 \times \frac{9}{100} + x \times \frac{3}{100}\right)$$

It is given that acid content in the resulting mixture must be more than 5% but less than 7% acid.

Therefore,

$$5\% \text{ of } (460 + x) < 460 \times \frac{9}{100} + \frac{3x}{100} < 7\% \text{ of } (460 + x)$$

$$\Rightarrow \frac{5}{100} \times (460 + x) < 460 \times \frac{9}{100} + \frac{3x}{100} < \frac{7}{100} \times (460 + x)$$

$$\Rightarrow 5 \times (460 + x) < 460 \times 9 + 3x < 7 \times (460 + x)$$

[multiplying by 100]

$$\Rightarrow 2300 + 5x < 4140 + 3x < 3220 + 7x$$

Taking first two inequalities,

$$2300 + 5x < 4140 + 3x$$

$\Rightarrow \quad 5x - 3x < 4140 - 2300$

$\Rightarrow \quad 2x < 1840$

$\Rightarrow \quad x < \frac{1840}{2}$

$\Rightarrow \quad x < 920$

Taking last two inequalities,

$4140 + 3x < 3220 + 7x$

$\Rightarrow \quad 3x - 7x < 3220 - 4140$

$\Rightarrow \quad -4x < -920$

$\Rightarrow \quad 4x > 920$

$\Rightarrow \quad x > \frac{920}{4}$

$\Rightarrow \quad x > 230$ ...(ii)

Hence, the number of litres of the 3% solution of acid must be more than 230 L and less than 920 L.

32. In drilling world's deepest hole it was found that the temperature T in degree celcius, x km below the earth's surface was given by $T = 30 + 25(x - 3)$, $3 \le x \le 15$. At what depth will the temperature be between 155°C and 205°C? [Delhi Gov. QB 2022]

Ans. $T = 30 + 25(x - 3)$, $3 \le x \le 15$

Where, T = temperature and x = depth inside the earth

The temperature should be between 155°C and 205°C

So, we get,

$\Rightarrow \quad 155 < T < 205$

$\Rightarrow \quad 155 < 30 + 25(x - 3) < 205$

$\Rightarrow \quad 155 < 30 + 25x - 75 < 205$

$\Rightarrow \quad 155 < 25x - 45 < 205$

Adding 45 to each term, we get

$\Rightarrow \quad 200 < 25x < 250$

Dividing each term by 25, we get

$\Rightarrow \quad 8 < x < 10$

Hence, temperature varies from 155°C to 205°C at a depth of 8 km to 10 km.

33. During preparation, a solution of 6% alcohol is to be diluted by adding 2% concentrated solution to it. The resulting mixture is to be more than 4% but less than 6% acid. If there is 660L of the 7% solution, how many litres of 2% solution will have to be added?

Ans. Let xL of 2% solution be added to 660L of 6% solution of acid.

Then, the total quantity of mixture = $(660 + x)L$

Total acid content in the $(660 + x)L$ of mixture

$= (660 \times \frac{6}{100} + x \times \frac{2}{100})$

It is given that acid content in the resulting mixture must be more than 4% but less than 7% acid.

Therefore,

$$4\% \text{ of } (660 + x) < 660 \times \frac{6}{100} + \frac{2x}{100} < 7\% \text{ of } (660 + x)$$

$$\Rightarrow \frac{4}{100} \times (660 + x) < 660 \times \frac{6}{100} + \frac{2x}{100} < \frac{7}{100} \times (660 + x)$$

$\Rightarrow \quad 4 \times (660 + x) < 660 \times 6 + 2x < 7 \times (660 + x)$ [multiplying by 100]

$\Rightarrow \quad 2640 + 4x < 3960 + 2x < 4620 + 7x$

Taking first two inequalities,

$2640 + 4x < 3960 + 2x$

$\Rightarrow \quad 4x - 2x < 3960 - 2640$

$\Rightarrow \quad 2x < 1320$

$\Rightarrow \quad x < \frac{1320}{2}$

$\Rightarrow \quad x < 660$

Taking last two inequalities,

$3960 + 2x < 4620 + 7x$

$\Rightarrow \quad -7x + 2x < 4620 - 3960$

$\Rightarrow \quad -5x < 660$

$\Rightarrow \quad x > \frac{660}{5}$

$\Rightarrow \quad x > 132$ ...(ii)

Hence, the number of litres of the 2% solution of acid must be more than 132 L and less than 710 L.